Optimal PMU Placement for Power System State Estimation

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***Abstract*— The Phasor Measurement Unit (PMU) is a commonly used measurement unit in power systems to perform state estimation, as it can measure the voltage and current phasors of bus systems. Although PMU placement can improve the accuracy of power system state estimation, it is expensive to place PMUs in every bus. In this study, an MDEP algorithm is proposed to achieve optimal PMU placement and reduce the number of PMUs while minimizing error. A Weighted Least Square (WLS) approach is implemented to estimate the power system state. The efficacy of the proposed algorithm is compared against previous algorithms in terms of meter placement and computation time.**

***Keywords*— MDEP algorithm, optimal PMU placement, WLS state estimation**

I. Introduction

In the modern era, the population is increasing every day, and along with it, the demand for electricity to support the daily activities of the public is also increasing. To address the challenge of supplying electricity to the public, state estimation is required to ensure the stability of the power system. This approach enables stable power supply without facing any power cuts. State estimation is used to precisely regulate the required generated power from the power system and provide it to the industry and residents. Additionally, state estimation acts as a security for the power system by preventing damage to the equipment of the power system when faults occur (Nagsarkar & Sukhija, 2014). Furthermore, the rounding values in calculations can cause errors in power system readings, in which state estimation can act as a correction mechanism for the value.

The Weighted Least Square (WLS) algorithm is one of the most extensively utilized algorithms for state estimation, developed in 1986 by Fred Schweppe. The WLS state estimation method employs three distinct state variables for different measurements, namely complex power flow, nodal voltage, and transformer ratio. The complex power flow utilizes the state variables of real and reactive power flow, while nodal voltage uses the voltage magnitude and voltage phase angle as the state variables. Furthermore, transformer turns ratio utilizes the transformer turn ratio angle and transformer turn ratio magnitude as the state variables. Different state variables are applied to respective measurements (Monticelli, 1999). For instance, all the three different state variables can be used for switch flow, zero impedance branch flow, and unknown impedance flow measurements.

Phasor Measurement Units (PMUs) are measurement units frequently utilized in the power grid, distribution line, and the transmission line to estimate the system state and monitoring processes in the power system. PMUs capture positive time-tagged measurements with high accuracy, collecting the voltage phasor and the current phasor value of the bus. Furthermore, PMUs can integrate with global positioning satellite (GPS) time stamps to deliver data to other transmission lines located in different points of location, enabling real-time monitoring in the power system. However, a major consideration in implementing PMUs in the power system is their cost. While PMUs have high accuracy for state estimation, implementing one in every bus of the power system is prohibitively expensive. Therefore, optimal PMU placement is necessary to minimize the number of PMUs used in the power system while maintaining state estimation accuracy at a high level (Yuill et al., 2011).

This study employed the proposed Modified Discrete Evolutionary Programming (MDEP) algorithm to optimize PMU placement and improve the accuracy of the WLS algorithm state estimation to reduce error considering observability. The simulation of the MDEP algorithm was tested on the IEEE 14 bus system, and its performance was compared with existing algorithms.

II. Observability of PMU Placement

A. Observability Analysis

The observability of PMU placement is critical to ensure the accuracy of state estimation in a bus system. In WLS algorithm, the total measurement must be more than $2n$-1 where n represents the total number of buses in the state vector to be estimated. The gain matrix was used in this study to signify the topological observability of the power system network. Improper PMU placement can result in singularity in the gain matrix and consequently cause an invalid calculation for state estimation.

The formation of a rank matrix occurs through the utilization of Gaussian elimination on the Jacobian H matrix (Shahriar et al., 2019). The rank of a matrix with a row echelon form is equivalent to the number of its nonzero rows. Moreover, a matrix will be identified as a rank matrix when its rank number exceeds the number of columns. Eq. (1) shows constraint of the observability for bus system.

$$rank\ (H) \geq 2n - 1 \qquad (1)$$

The Jacobian H matrix in Eq. (2) is composed of both real and reactive power flow as well as real and reactive power injection. By taking the derivative of the real power injection and power flow with respect to the angle, a model can be formed to ascertain the observability of the bus system.

$$H_{P\theta} = \begin{bmatrix} \frac{\partial P_i}{\partial \theta_i} \\ \frac{\partial P_j}{\partial \theta_j} \\ \frac{\partial P_{ij}}{\partial \theta_i} \\ \frac{\partial P_{ij}}{\partial \theta_j} \end{bmatrix} \quad (2)$$

By solely considering the real power injection and flow and utilizing Eq. (1), one can rephrase the observability as shown in Eq. (3), where only the derivatives of the real power injection and flow are considered in determining the observability of the bus system.

$$rank\ (H_{P\theta}) \geq n - 1 \quad (3)$$

III. MDEP Algorithm for Optimal PMU Placement

The MDEP algorithm is an improvement on the DEP algorithm as shown in Fig. 1. Initially, the MDEP algorithm generates a $d \times d$ matrix population, as illustrated in Table I. The dimension of the generated matrix depends on the total number of buses in the bus system. This population is then subject to the constraint, which determines the observability of the bus arrangement. Once observable buses are identified, they are used to calculate the state estimation error.

To update the particles' position, the observable bus undergoes Mutation 1 process, in which a nonidentical value replaces the diagonal element of the buses, as presented in Table II. Next, the observable population of the bus undergoes the Mutation 2 process, in which the left triangular element of the buses is removed, as shown in Table III. The purpose of this mutation is to minimize the population of the buses. Subsequently, the new bus arrangement undergoes a constraint to assess its observability after the second mutation. The observable bus arrangement is then used to calculate the state estimation error.

These observable buses are then combined to undergo a ranking and selection process. The bus arrangement with the lowest state estimation error is ranked at the top, while the opposite occurs at the bottom. The mutation process is iterated until a converged result is obtained, when the generated optimal bus arrangement differs from the original one. Thus, the MDEP algorithm not only obtains the observable bus for PMU placement but also minimizes the number of PMU placements required for power system state estimation.

TABLE I. INITIAL POPULATION

Intial population				
$x_{1,1}$	$x_{1,2}$	$x_{1,3}$	…	$x_{1,d}$
$x_{2,1}$	$x_{2,2}$	$x_{2,3}$	…	$x_{2,d}$
$x_{3,1}$	$x_{3,2}$	$x_{3,3}$	…	$x_{3,d}$
⋮	…	⋱	…	⋮
$x_{d,1}$	$x_{d,2}$	$x_{d,3}$	…	$x_{d,d}$

TABLE II. MATRIX OF MUTATION 1

Mutation 1				
$x_{rand,1}$	$x_{1,2}$	$x_{1,3}$	…	$x_{1,d}$
$x_{2,1}$	$x_{rand,2}$	$x_{2,3}$	…	$x_{2,d}$
$x_{3,1}$	$x_{3,2}$	$x_{rand,3}$	…	$x_{3,d}$
⋮	…	⋱	$x_{rand,\dots}$	⋮
$x_{d,1}$	$x_{d,2}$	$x_{d,3}$	…	$x_{rand,d}$

TABLE III. MATRIX OF MUTATION 2

Mutation 2				
$x_{rand,1}$	$x_{1,2}$	$x_{1,3}$	…	$x_{1,d}$
-	$x_{rand,2}$	$x_{2,3}$	…	$x_{2,d}$
-	-	$x_{rand,3}$	…	$x_{3,d}$
-	-	-	$x_{rand,\dots}$	⋮
-	-	-	-	$x_{rand,d}$

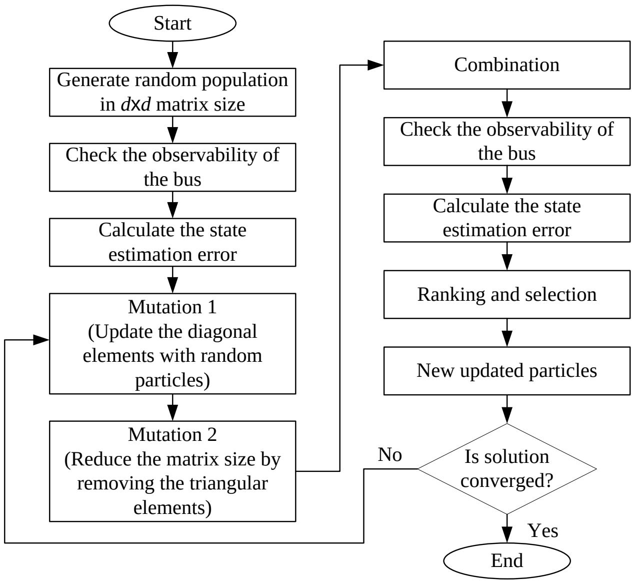

Fig. 1. Flowchart of MDEP algorithm

B. *WLS Algorithm*

The WLS algorithm is considered to compute the state estimation which is conducted in the IEEE bus test system. Figure 3 depicts the flowchart of the WLS algorithm. Initially, the state vector and measurement vector must be identified. These vectors are necessary for the WLS algorithm to calculate the measurement vector error, denoted as $\boldsymbol{\xi}$ in Eq. (4). This error is calculated using the nonlinear function vector, $h(x)$, which relates the state vector, x as in Eq. (5) and the measured vector, z, which is obtained from PMU placement in the bus system as in Eq. (6). The state vector consists of voltage magnitude and phase angle of each bus.

$$\boldsymbol{\xi} = \boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{z} \quad (4)$$

$$\boldsymbol{x} = [\theta_2, \theta_3, \theta_4, \dots, \theta_n,\ V_1, V_2, V_3, \dots, V_n] \quad (5)$$

$$\boldsymbol{Z} = \begin{bmatrix} V_i^{\,a} \\ P_i^{\,a} \\ Q_i^{\,a} \\ P_i^{\,ab} \\ Q_i^{\,ab} \end{bmatrix} \quad (6)$$

where i = number of iterations
$V_i^{\,a}$ = voltage at bus a
$P_i^{\,a}$ = real power injection at bus a
$Q_i^{\,a}$ = reactive power injection at bus a
$P_i^{\,ab}$ = real power flow from bus a to b
$Q_i^{\,ab}$ = reactive power flow from bus a to b

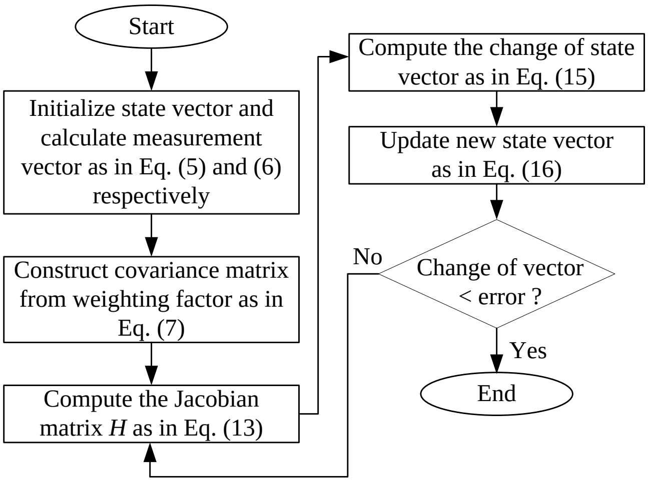

Fig. 2. Flowchart of WLS algorithm

The measurement variables comprise real and reactive power injection, real and reactive power flow, voltage magnitude and phase angle. The iteration count, denoted as i, is initially set to 1. The algorithm then uses the weighting factor to construct a covariance matrix. A diagonal covariance matrix is formed when errors are assumed to be independent, as shown in Eq. 7. The minimization of the objective function in the WLS state estimation is solved using Eq. 8.

$$R = \begin{bmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_d^2 \end{bmatrix} \quad (7)$$

$$\boldsymbol{J}(\boldsymbol{x}) = \sum_{i=1}^{m} \frac{[z_i - h(x)_i]^2}{\mathrm{R}_{ii}} \quad (8)$$

where z_i = measured value in i^{th}
$h(x)_i$ = nonlinear function vector that relates to the states vector, x
R_{ii} = measured error value in the i^{th} of the covariance matrix.

The Jacobian H matrix is formed using the state vector. Fig. 4 illustrates the two-port π-model for a network branch, which is utilized to compute the real and reactive power flow and the real and reactive power injection in the nonlinear function $h(x)$. The real and reactive power injection are calculated using Eq. 9 and Eq. 10 respectively whereas the real and reactive power injection is calculated using Eq. 11 and Eq. 12 respectively.

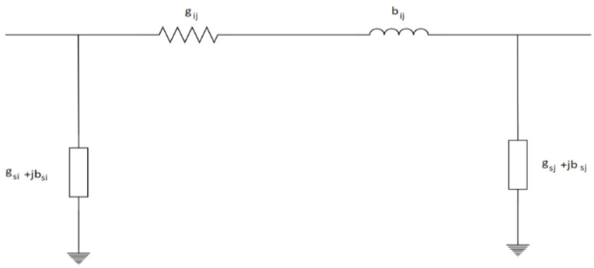

Fig. 3. Two port π-model for network branch

$$P_F = V_i^{\,2}\left(g_{ij} + g_{si}\right) - V_i V_j \left(g_{ij}cos\theta_{ij} + b_{ij}sin\theta_{ij}\right) \quad (9)$$

$$Q_F = -V_i^{\,2}\left(b_{ij} + b_{si}\right) - V_i V_j \left(g_{ij}sin\theta_{ij} + b_{ij}cos\theta_{ij}\right) \quad (10)$$

$$P_i = V_i \textstyle\sum_{j\in N_i} V_j \left(G_{ij}cos\theta_{ij} + B_{ij}sin\theta_{ij}\right) \quad (11)$$

$$Q_i = V_i \textstyle\sum_{j\in N_i} V_j \left(G_{ij}sin\theta_{ij} + B_{ij}cos\theta_{ij}\right) \quad (12)$$

where g_{ij}, b_{ij} = the series branch admittance connecting to bus i and j
g_{si}, b_{si} = the shunt branch admittance connecting to bus i and j
G_{ij}, B_{ij} = the complex bus admittance matrix's ij^{th} element
θ_{ij} = voltage angle difference between bus i and j
N_i = a group of bus numbers that are linked to bus i

The Jacobian matrix, $\boldsymbol{H}$, is formed according to Eq. 13. The left-hand side elements of the matrix consists of the partial derivative with respect to the angle, while the right-hand side elements consists of the partial derivative with respect to the voltage. The gain matrix is be calculated using Eq. 14 after the Jacobian matrix, $\boldsymbol{H}$, has been obtained. The gain matrix is used in Eq. 15 to calculate the change of the state variable for the variables to be updated.

$$\boldsymbol{H} = \begin{bmatrix} 0 & \frac{\partial \mathrm{Vm}}{\partial \mathrm{V}} \\ \frac{\partial P_{inj}}{\partial \theta} & \frac{\partial P_{inj}}{\partial \mathrm{V}} \\ \frac{\partial Q_{inj}}{\partial \theta} & \frac{\partial Q_{inj}}{\partial \mathrm{V}} \\ \frac{\partial P_F}{\partial \theta} & \frac{\partial P_F}{\partial \mathrm{V}} \\ \frac{\partial Q_F}{\partial \theta} & \frac{\partial Q_F}{\partial \mathrm{V}} \end{bmatrix} \quad (13)$$

$$\boldsymbol{G} = \boldsymbol{H^T R^{-1} H} \quad (14)$$

$$\Delta \boldsymbol{x}^k = \boldsymbol{H}^T \boldsymbol{R}^{-1}[\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}^k)]\boldsymbol{G}(\boldsymbol{x}^k)^{-1} \quad (15)$$

where k = number of iterations
$\boldsymbol{H}^T$ = transpose of the Jacobian H matrix
$\boldsymbol{h}(\boldsymbol{x}^k)$ = nonlinear function vector that relates the states vector, x to the measurement, Z_i
$\boldsymbol{G}(\boldsymbol{x}^k)$ = gain matrix

Next, the algorithm uses the change of state vector to calculate the new state vector, as illustrated in Eq. 16. This new state vector is then updated in the algorithm. The algorithm is iteratively computed until the change of state vector is less than the tolerance value.

$$x^{k+1} = x^k + \Delta x^k \tag{16}$$

IV. Results

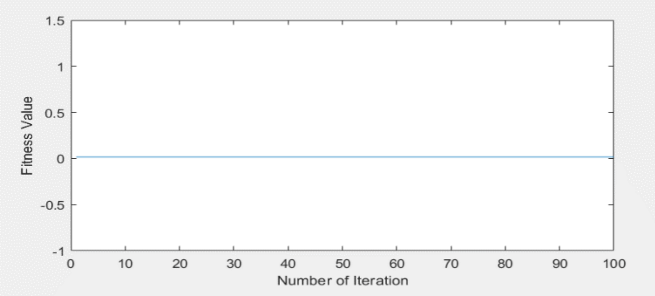

Fig. 4. MDEP algorithm convergence characteristic

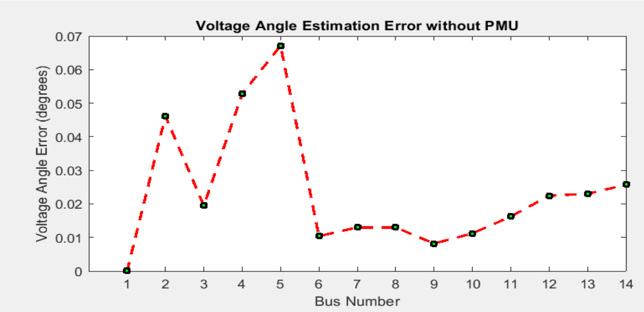

Fig. 5. Phase angle error by state estimation

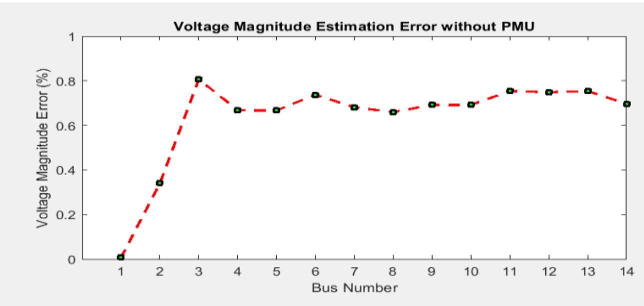

Fig. 6. Voltage magnitude error by state estimation

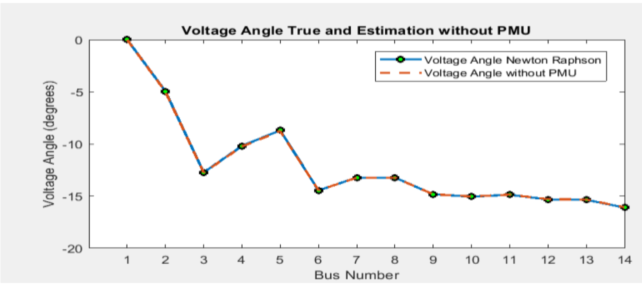

Fig. 7. Phase angle by state estimation compared with Newton Raphson load flow analysis

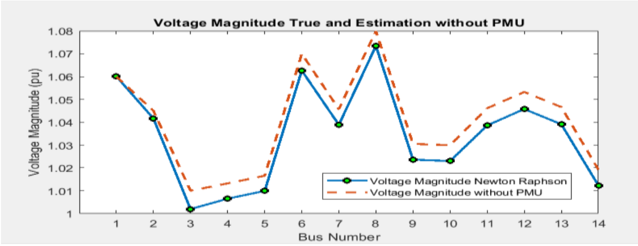

Fig. 8. Voltage magnitude by state estimation compared with Newton Raphson load flow analysis

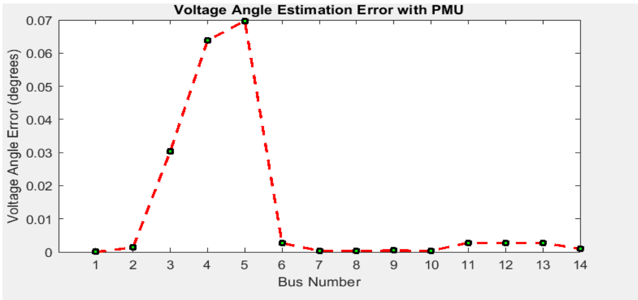

Fig. 9. Phase angle error by state estimation using optimal PMU placement

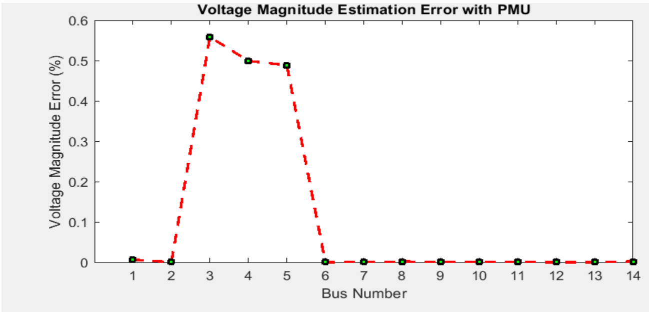

Fig. 10. Voltage magnitude error by state estimation using optimal PMU placement

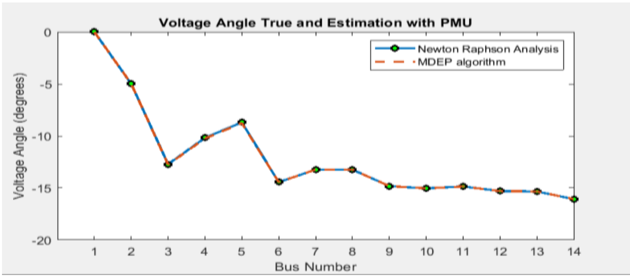

Fig. 11. Phase angle by state estimation using optimal PMU placement compared with Newton Raphson load flow analysis

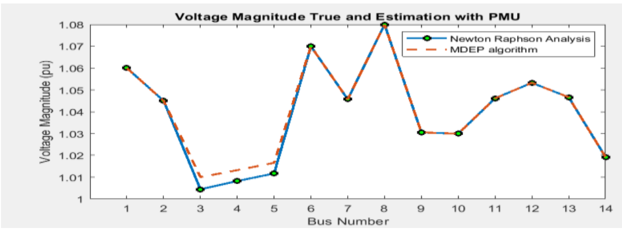

Fig. 12. Voltage magnitude by state estimation using optimal PMU placement compared with Newton Raphson load flow analysis

TABLE IV. COMPARISON OF ALGORITHM

Algorithm	Number of PMU placement (Bus)	Computation time (s)
DEP	4	0.29
MDEP	4	0.56
DeFS [5]	6	0.094
IPSO [6]	3	2.31
PSO [6]	3	2.52

V. DISCUSSION

Fig. 4 illustrates the convergence of the MDEP algorithm with minimum error of 0.0164 in the state estimation. Optimal PMU placement was identified to be at Buses 9, 6, 7, and 2. Fig. 5 and 6 illustrate the phase angle and voltage magnitude errors by the state estimation without optimal PMU placement. The maximum error for phase angle was identified to be at Bus 5 while the maximum error for voltage magnitude was identified to be at Bus 3.

A comparison was made between the state estimation and the Newton Raphson method for phase angle and voltage magnitude as illustrated in Fig. 7 and 8 respectively. Bus 2 to Bus 14 shows a small error in phase angle ranging from 0.01 to 0.07. It shows that this phase angle error does not result in a significant mismatch with the Newton Raphson method. On contrary, voltage magnitude shows a significant error which ranging from 0.38 to 0.8 from Bus 2 to Bus 14. Thus, the voltage magnitude shows a significant mismatch with the Newton Raphson method.

The optimal PMU placement, determined by the MDEP algorithm, included buses 9, 6, 7, and 2. This placement was subsequently implemented in the WLS state estimation. Fig. 9 depicts a significant reduction in phase angle error for all buses except buses 3, 4, and 5, in comparison to Fig. 7 with the values ranging between 0.003 to 0.07. Similarly, Fig. 10 shows a significant reduction in voltage magnitude error for all buses except buses 3, 4, and 5, in comparison to Fig. 8 with the values ranging between 0.5 to 0.57. These results indicate a considerable improvement in state estimation accuracy resulting from the PMU placement.

To prove the accuracy of the state estimation based on optimal PMU placement, a comparison was made to Newton Raphson method for phase angle and voltage magnitude as depicted in Fig. 11 and 12 respectively. The results for both phase angle and voltage magnitude shows no significant error between state estimation and Newton Raphson method. Therefore, it is proven that the optimal PMU placement by MDEP algorithm improve the accuracy of state estimation.

Additionally, the performance of MDEP algorithm was compared with other algorithms such as Discrete Evolutionary Programming (DEP), Differential Evolution Based Feature Subset (DeFS), Improved Particle Swarm Optimization (IPSO), and Particle Swarm Optimization (PSO) in terms of computation time and number of PMU placement as presented in Table IV. The DeFs algorithm identified a greater number of PMU placement as compared with MDEP although it can achieve the most minimum computation time. However, a greater number of PMU placements leads to increased project costs.

Accordingly, the PSO and IPSO algorithms identified the minimum number of PMU placements with nearly identical computation times, however, longer duration in comparison with MDEP. While over-reducing PMU placement and increasing computation time may not significantly impact small-scale bus systems such as the IEEE 9 bus and IEEE 14 bus systems, the same approach implemented in a larger-scale bus system such as the IEEE 118 bus system will likely result in significant computation time that is unsuitable for state estimation given that dynamic data collected from actual bus systems requires a timely computation to avoid errors in state estimation. Hence, the performance of MDEP for optimal PMU placement was comparable with previous optimization techniques.

VI. CONCLUSION

In conclusion, the proposed algorithm has successfully reduced the number of the PMU placement in the IEEE 14 bus system to four buses. The performance of the MDEP algorithm was compared against other optimization algorithms, with MDEP showcasing superior overall performance, requiring less computing time, and achieving minimum number of PMU placements.

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